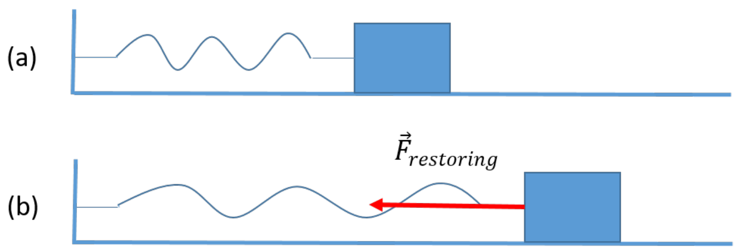
SIMPLE HARMONIC MOTION

Created by: Binh Cao

- **Simple harmonic motion** (SHM) is a type of periodic motion. Two simple systems of SHM that are mainly discussed in college are **an ideal spring** and **a simple pendulum**. But before discussing these 2 systems, it is essential to go over periodic motion first.
- **Periodic motion** or **oscillation** refers to kinds of motion that repeat themselves over and over. For example, a clock pendulum.
- Understanding periodic motion is critical for understanding more complicated concepts like mechanical waves (e.g. sound) and electromagnetic waves (e.g. light)
- Oscillation is characterized by an **equilibrium** and a **restoring force**. At equilibrium, restoring force on the object is zero. When the object is displaced from equilibrium, a restoring force acts on the object to restore its equilibrium. For example, a spring.



(a) The object is at equilibrium. The spring is neither stretched nor compressed.

(b) The object is displaced and the spring is stretched.

- When the restoring force is directly proportional to displacement from equilibrium, the oscillation is called **simple harmonic motion** (SHM).
- Important characteristics of any periodic motion:
  - **Amplitude** (A) is maximum magnitude of displacement from equilibrium
  - **Period** (T) is the time to complete one cycle (unit: s)
  - **Frequency** (f) is the number of cycles in a unit of time (unit: s^{-1})
  - Period and frequency are related by the following relationship:

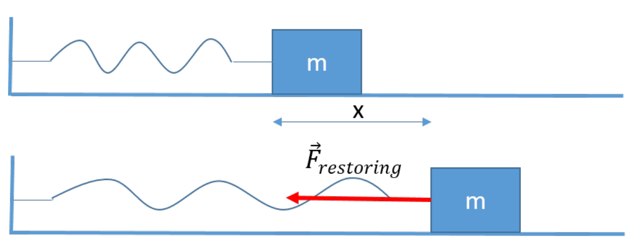
$$T = \frac{1}{f} \; or \; f = \frac{1}{T}$$

  - **Angular frequency** (ω):

$$\omega = 2\pi f = \frac{2\pi}{T} \text{ (rad/s)}$$

A/ Ideal spring

$$F_{restoring} = -kx$$



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k: spring constant

x: displacement from equilibrium

$\omega = \sqrt{\frac{k}{m}}; f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}; T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$x = A\cos(\omega t + \Phi)$

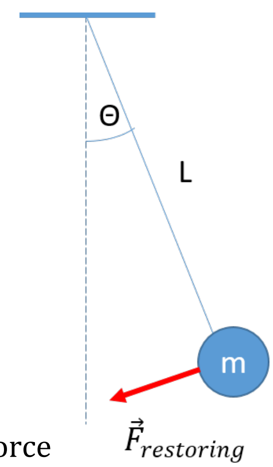
Φ: initial angular displacement

$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = const$

E: mechanical energy of the system

v_x: velocity of mass m at x (m/s)

B/ Simple pendulum



$F_{restoring} = -mgsin\theta \cong -mg\theta$ (when θ is small)

Θ: angular displacement form equilibrium

$\omega = \sqrt{\frac{g}{L}}; f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}; T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

L: string length (m)

C/ Examples

1/ When a body of unknown mass is attached to an ideal spring with force constant 120 N/m, it is found to vibrate with a frequency of 6.00 Hz. Find

(a) The period of the motion;

(b) The angular frequency;

(c) The mass of the body.

Solution:

k = 120 N/m; f = 6.00 Hz

(a) $T = \frac{1}{f} = \frac{1}{6.00} = 0.167\ s$

(b) $\omega = 2\pi f = 2\pi \times 6.00 = 37.7$ (rad/s)

(c) $\omega = \sqrt{\frac{k}{m}}$ or $m = \frac{k}{\omega^2} = \frac{120}{37.7^2} = 0.0845\ kg = 84.5\ g$

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2/ A building in San Francisco has light fixtures consisting of small 2.35-kg bulbs with shades hanging from the ceiling at the end of light, thin cords 1.50 m long. If a minor earth quake occurs, how many swings per second will these fixtures make?

<u>Solution:</u>

m = 2.35 kg; L = 1.50 m

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8}{1.50}} = 0.407\ swings/s$$

D/ Practice problems (Answer key is below)

1/ A 1.50-kg mass on a spring has displacement as a function of time given by the equation

$$x(t) = (7.40cm)\cos[(4.16\ s^{-1})t - 2.42]$$

Find

(a) The time for one complete vibration;

(b) The force constant of the spring;

(c) The maximum speed of the mass;

(d) The maximum force on the mass;

(e) The position, speed and acceleration of the mass at t = 1.00 s;

(f) The force on the mass at that time

(g) The mechanical energy of the system

2/ After landing on an unfamiliar planet, a space explorer constructs a simple pendulum of length 50.0 cm. She finds that the pendulum makes 100 complete swings in 136 s. What is the value of g on this planet?

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Reference:

Kramer, Laird.Young, Hugh D. (2012) *Study guide, Sears & Zemansky's University physics, 13th edition, Young and Freedman* /San Francisco, CA : Pearson,

Answer key

1/ a) 1.51 s; b) 26.9 N/m; c) 30.8 cm/s; d) 1.92 N; e) -0.0125m, 30.4 cm/s, 0.216 m/s^2; f) 0.324 N; g) 0.0737 J

2/ 10.7 m/s^2